CYPRESS MICROSYSTEMS

Application Note AN2031

Adjustable Sallen and Key Low-Pass Filters

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Summary
Sallen and Key low-pass filters are constructed using continuous time (CT) analog PSoC™ blocks. Methods are shown for making the filters adjustable for use as audio-tone control.

Introduction
The PSoC device has switched capacitor (SC) analog blocks that can be used to form filters of most common types. However, these may not always be suitable for all filter needs. In this case, the continuous time analog blocks can be utilized to form standard classes of analog filters.

The Need for Non-SC Filters
PSoC SC filter User Modules have characteristics that are determined by the ratios of capacitor values in the selected blocks and the clock frequency driving the blocks. These filters are useful at frequencies up to the Nyquist rate, or half of the sampling frequency.

The highest ratio of sampling frequency to filter nominal corner frequency (over sample ratio, or OSR) is limited by available capacitor ratios to approximately 160. Thus, the maximum usable frequency for a low-pass filter is 80 times the corner frequency. There may be a need for attenuation of noise and spurious signals at frequencies above this range; or, the system design may have simply consumed all SC blocks and an anti-aliasing filter is still needed. Accordingly, we need another way to make the low-pass filter.

Low-Pass Filter Transfer Function
The frequency performance of the two-pole, low-pass filter is described by the transfer function:

$$\frac{Vout}{Vin} = \frac{K\omega_n^2\omega_0^2}{s^2 + sd\omega_n\omega_0 + \omega_n^2\omega_0^2}$$

where: ω_0 = nominal corner frequency,
ω_n = normalized corner frequency ($2\pi f_c$),
d = damping factor.

ω_0 and d for standard filter implementations are calculated from the complex pole locations for the specific filter to be implemented. The complex pole locations $(\alpha + j\beta)$ for a low-pass filter are converted to ω_0 and d.

$$\omega_0 = \sqrt{\alpha^2 + \beta^2} \qquad d = \frac{2\alpha}{\omega_0}$$

A design example is shown for a Butterworth filter. The transfer function for the Butterworth has ω_0 = 1.0 and d = *1.414.* Filters other than Butterworth will have ω_0 shifted from 1.0. Filters with flatter in-band response, such as Chebychev, have lower damping and slightly lower ω_0. Pole locations, or ω_0 and d, are readily available from any number of filter references[1].

[1] Electronic Filter Design Handbook, Arthur B. Williams and Fred J. Taylor, McGraw-Hill, 1981, Tables in Chapter 11.

Sallen and Key Low-Pass Filters

The Sallen and Key[2] filter implementation is the simplest active filter. It uses the lowest number of components and the design equations are relatively straightforward. It has been discussed, reviewed, and analyzed in great depth in hundreds of published articles, reference texts, design handbooks[3] and at great length on the web since the original Sallen and Key paper in the IRE in 1955. This form is often preferred to the biquad and multiple-loop feedback approaches because it does not invert the signal.

The simplest form of the Sallen and Key implementation of the two-pole, low-pass filter is shown in Figure 1. Multiple pole-pair filters are easily implemented by cascading filter sections.

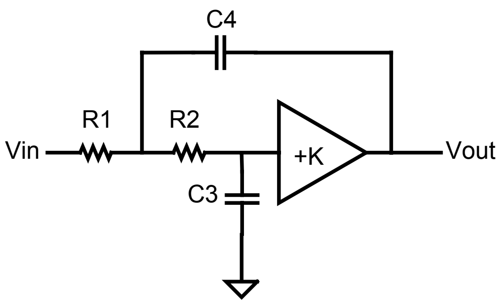

Figure 1: Schematic Low-Pass Filter

The circuit has the transfer function:

$$\frac{Vout}{Vin} = \frac{\frac{K}{R_1 R_2 C_3 C_4}}{s^2 + s\left(\frac{1-K}{R_2 C_3} + \frac{R_1 + R_2}{R_2 R_2 C_4}\right) + \frac{1}{R_1 R_2 C_3 C_4}}$$

From this, we derive design equations:

$$R_1 = \frac{d + \sqrt{d^2 - 4\left(C_3/C_4 + 1 - K\right)}}{2\omega_0 \omega_n (C_3 + (1-K)C_4)}$$

$$R_2 = \frac{1}{R_1 C_3 C_4 \left(\omega_0 \omega_n\right)^2}$$

C3 can be arbitrarily chosen to meet the impedance requirements of the circuit.

C4 must be chosen to meet the requirement:

$$d^2 - 4\left(C_3/C_4 + 1 - K\right) \geq 0$$

or $$C_4 \geq \frac{C_3}{d^2/4 \;\; - 1 + K}$$

In the simplest case, setting the low-pass gain, K, to 1.00 yields a further simplification of the calculation of C4:

$$C_4 \geq \frac{4C_3}{d^2}$$

Once the value for C4 has been established, finding R1 and R2 is a simple calculation.

1. PSoC Design Example

A design example is demonstrated for two-pole, low-pass Butterworth at 1000 Hz. In this case:

$$d = 1.414$$
$$\omega_0 = 1.00$$
$$\omega_n = 2\pi 1000$$
$$K = 1.00$$

Let $C_3 = 0.001\mu F$

$$C_4 \geq \frac{4 \cdot 0.001\mu F}{1.414^2} \geq .002\mu F$$

Let $C_4 = 0.0022\mu F$

$$\left(1 - K\right) = 0$$

so…

$$R_1 = \frac{d + \sqrt{d^2 - 4(\frac{C_3}{C_4})}}{2\left(C_3 + C_4(0)\right)\omega_0 \omega_n}$$

$$R_1 = \frac{1.414 + 1.414}{2 * 0.001uF * 1.0 * 2\pi 1000} = 146.5k\Omega$$

$$R_2 = \frac{1}{146.5k\Omega(.001uF)(.0022uF)} = 78.67k\Omega$$

The result is a filter that follows the canonical form for the two-pole, low-pass Butterworth filter.

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2 Sallen, R.P. "A Practical Method of Designing RC Active Filters," IRE Transactions Circuit Theory, Vol CT-2, March 1955, pp74-85.

3 Active Filter Cookbook, Don Lancaster, Howard W. Sams & Co., 1975.

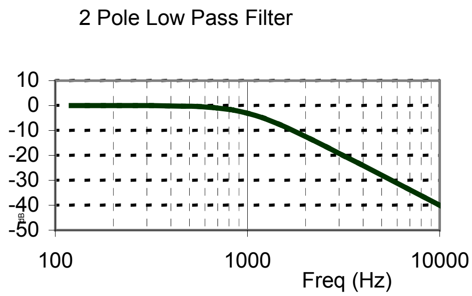

Figure 2: 1000 Hz Low-Pass Filter

Accuracy of the filter is limited primarily by the accuracy of the selected passive components.

PSoC Implementation

The gain stage (K) for the filter is a continuous time (CT) Programmable Gain Amplifier (PGA) User Module selectable in PSoC Designer.

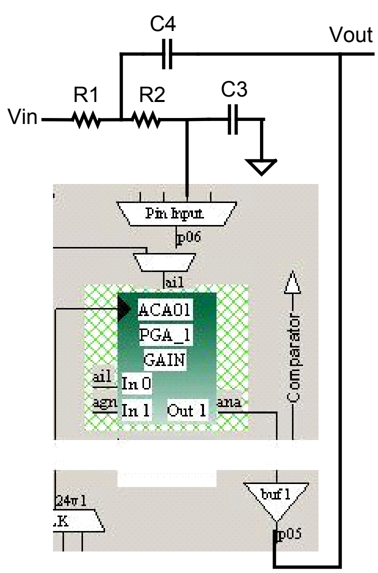

Figure 3: Low-Pass Filter Using PGA

The PGA output is enabled onto the Analog Output Bus for the column. The Analog Output Bus drives an analog buffer, which is connected to an output on Port 0. This port output drives feedback capacitor, C4. C3 should be connected to analog ground. There are several selections for this connection:

- User-provided external ground, or
- Buffered analog ground, generated from AGND signal in the CT block and passed through that blocks' TestMux to another analog buffer, or
- Resistive divider from V_{cc} to V_{ss}, where both resistors are 2.0 times the nominal value for R2.

The resulting output in all cases is referenced to the selected PSoC analog ground.

This combination of the PGA and the analog buffer has sufficient gain-bandwidth to provide accurate low-pass filters of audio signals. This filter functions well for roll-off frequencies up to one tenth the gain bandwidth of the amplifier. The power level for PGA should be set to match the upper-end bandwidth requirements of the filter.

Adjustable Low-Pass Filter

A simple modification to the filter block diagram allows a degree of flexibility. The addition of a feedback gain amplifier is shown:

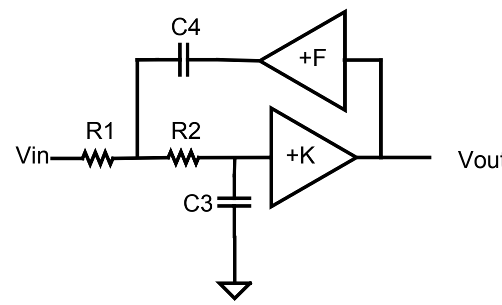

Figure 4: Low-Pass Filter Schematic with Adjustable Feedback

The transfer function is only slightly more complex, and the design equations are identical for feedback gain, F=1.00.

$$\frac{Vout}{Vin} = \frac{\dfrac{FK}{R_1 R_2 C_3 C_4}}{s^2 + s\left(\dfrac{1-FK}{R_2 C_3} + \dfrac{R_1 + R_2}{R_1 R_2 C_4}\right) + \dfrac{1}{R_1 R_2 C_3 C_4}}$$

The transfer function is valid as long as:

$$\left(\frac{1-FK}{R_2C_3}+\frac{R_1+R_2}{R_1R_2C_4}\right)>0$$

When this term goes negative, the poles shift from complex conjugate pair, with both having negative real part, to a pair of real poles, but with one pole having positive real part. This is, of course, an oscillator.

2. PSoC Design Example

We build on the structure of the earlier example to configure the adjustable filter. The design equations are the same, but an additional PGA is shown to implement the feedback gain, F.

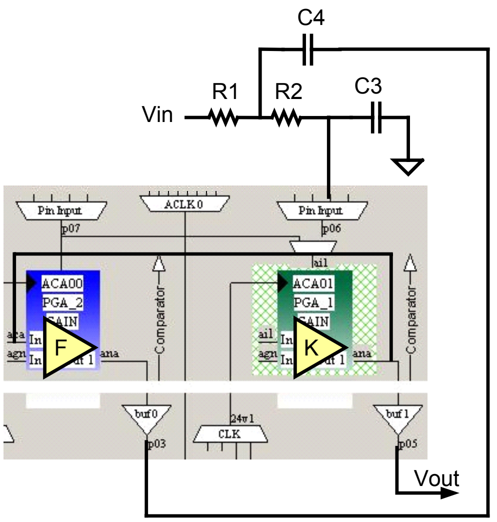

Figure 5: Low-Pass Filter with Second PGA for Adjustable Damping

The output of PGA K (in ACA01) is routed in the PSoC connections to the input of the adjacent PGA (in ACA00) to form the feedback amplifier, F. The output of F is connected to its Analog Output Bus and the analog buffer is enabled. The output of this buffer drives R4. The output of PGA K does not need to be connected to its Analog Output Bus unless it is required for test purposes to bring the signal off-chip. The output of PGA K can be directly connected to a PGA in an adjacent CT block (ACA02 in this example), the input of a switched capacitor (SC) low-pass filter (LPF2), SC band-pass filter (BPF2), or analog-to-digital converter.

Using the 150 Hz Butterworth example as before, we show the range of control in damping factor as a function of feedback gain.

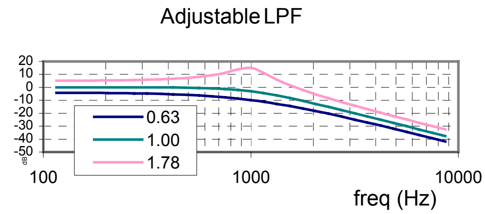

Figure 6: 150 Hz Low-Pass Filter Adjustable Damping

The lower limit of feedback gain is the 0.063 setting of the gain for PGA F. The upper limit for feedback gain is determined by transfer function requirements as discussed earlier. This configuration allows adjustment of the response at the nominal corner frequency over a range from -6 to +12 dB. This range is useful for audio-tone controls or dynamic adjustment of closed-loop, feedback-control transfer function.

The feedback gain can be adjusted "on-the-fly" using the SetGain_xxx commands in the API for the PGA F User Module.

A simple spreadsheet for the design of this filter is included in the documentation section of PSoC Designer, releases 3.0 and later.

Summary

Complete design methodology has been demonstrated for building Sallen and Key low-pass filters using the analog capabilities of the PSoC device.

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